A Firepower Assignment Method for Ship-to-air Missile System Based on Hybrid Genetic Algorithm

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Abstract

Aiming at the issue of constrained optimization problem of firepower assignment for ship-to-air missile system, hybrid genetic algorithm has been proposed in this paper based on adaptive penalty function. The constrained optimization problem is converted into the unconstrained one through the introduction of adaptive penalty function. Hybrid genetic algorithm is proposed by combining standard genetic algorithm with hill climbing to solve the unconstrained optimization problem, which can get global optimization results of the firepower assignment, and provide decision support for the firepower assignment.

Keywords

Firepower Assignment; Ship-to-air Missile System; Hybrid Genetic Algorithm

Introduction

Suppose that ship-to-air missile system uses semi-active radar homing, the firepower assignment for the system is to assign the illumination radars of the system to the arriving targets. The ship-to-air missile fire effect is affected by the solution quality of the assignment scheme.

The firepower assignment is a typical integer programming problem and belongs to 0-1 Programming problem. Genetic algorithm a searching arithmetic of global optimization (Mitchell Melanie, 1999), based on principles inspired by natural genetics "survival of the fittest, excellent win and inferior discard", can be used to solve the firepower assignment. Previous work used standard genetic algorithm to solve the optimization problem of the firepower assignment for the system (Youtang Tong, 2009). However, standard genetic algorithm is of insufficient local optimization, in contrast to hill climbing (Kenekayoro Patrick, 2012), and the two methods can be combined with each other to form

hybrid genetic algorithm that can be proposed to get global optimization results.

This paper proposes the firepower assignment model for the system by considering commander's decision and the illumination radar coverage as constraints. Afterwards, hybrid genetic algorithm based on adaptive penalty function is used to solve the constrained optimization problem of the firepower assignment.

Firepower Assignment Model

It is supposed that there are N arriving targets, and the system has M illumination radars, M≥1. Ensuring maximal targets to meet the illumination radar irradiation conditions is considered as optimization objective (Yang Zhang, 2012), and then supposing that target number which meet the illumination radar irradiation conditions is defined by f whose maximization is the key of the firepower assignment problem.

$$\max f = \sum_{i=1}^{M} \sum_{j=1}^{N} R_j X_{ij} \tag{1}$$

$$g_j = \sum_{i=1}^{M} X_{ij} - b_j \leq 0 \tag{2}$$

$$g_{N+i} = \sum_{j=1}^{N} X_{ij} - 1 \leq 0 \tag{3}$$

where R_j is the j-th target threat value, X_{ij} is a Boolean value used to determine whether the i-th illumination radar is assigned to the j-th target. If the i-th illumination radar is assigned to the j-th target, then $X_{ij}=1$, otherwise $X_{ij}=0$. Eq. (2) represents that the number of illumination radar assigned to the j-th target doesn't exceed b_j. Eq. (3) represents that the i-th illumination radar is assigned to at most one target.

The constraints of the firepower assignment model mainly include commander's decision and the

illumination radar coverage.

Commander's Decision Constraint

Because of the complex campaign circumstances, the achieved data is impossible to be complete, determinate and reliable. Commander's decision is necessary, namely, the commander should have definite interference on the firepower assignment scheme. If the j-th target is of very importance, the R_j value of the j-th target given by the commander's decision can have significant impact on the firepower assignment scheme.

Illumination Radar Coverage Constraint

For single illumination radar (let A radar), only when the position of a single target is within the A radar coverage, can the target be illuminated by the A radar.

Let the time for the first target to reach the long-range limit of the A radar coverage be $T_Y(A,1)$, and the time for the first target to stay within the A radar coverage be $T_D(A,1)$, and the time for the first target to be initially illuminated by the A radar be $T_L(A,1)$. The first target can be effectively illuminated by the A radar by satisfying Eq. (4).

$$T_Y(A,1) \leq T_L(A,1) \leq T_Y(A,1) + T_D(A,1) \qquad (4)$$

After the completion of the illumination task of the first target, the second target is illuminated by the A radar which requires a transfer time to initialize work. Let the minimum transfer time for the A radar be T_Z, and then the second target can be effectively illuminated by the A radar by satisfying Eq. (5) and Eq. (6).

$$T_Y(A,2) \leq T_L(A,2) \leq T_Y(A,2) + T_D(A,2) \qquad (5)$$

$$T_Z \leq T_L(A,2) - T_L(A,1) \qquad (6)$$

where $T_Y(A,2)$ is the time for the second target to reach the long-range limit of the A radar coverage, $T_D(A,2)$ is the time for the second target to stay within the A radar coverage, $T_L(A,2)$ is the time for the second target to be initially illuminated by the A radar.

Hybrid Genetic Algorithm Based on Adaptive Penalty Function

Fitness Function Design

The firepower assignment model is a constrained optimization problem which can't be directly solved by genetic algorithm. By introducing the penalty function, the constrained optimization problem is converted into an unconstrained optimization problem which can be solved by genetic algorithm.

This paper uses adaptive penalty function (Qiongsun Liu etc, 2006), and the penalty factor is a function of the independent variable X_{ij}. Meanwhile, in order to accelerate the convergence speed, this paper draws a "multi-level punishment" thinking which gives bigger constraint violation severer punishment, and smaller constraint violation lesser punishment. The fitness function can be expressed as in Eq. (7).

$$eval = f + C \cdot p \qquad (7)$$

$$p = \sum_{k=1}^{M+N} Q_k \cdot \max\{0, g_k\} \qquad (8)$$

$$Q_k = \frac{\max\{0, g_k\}}{\sum_{l=1}^{M+N} \max\{0, g_l\}} \qquad (9)$$

$$C = 1 + \frac{|f|}{1+p} \qquad (10)$$

where p is penalty function, C is the penalty function coefficient.

Hybrid Genetic Algorithm Design

Firepower assignment method for ship-to-air missile system based on hybrid genetic algorithm is depicted in Fig.1. The method generates the better individual with the complete global search when the termination condition is met. Then through the use of hill climbing, the method continues to complete the local search process, and best individual is ultimately got.
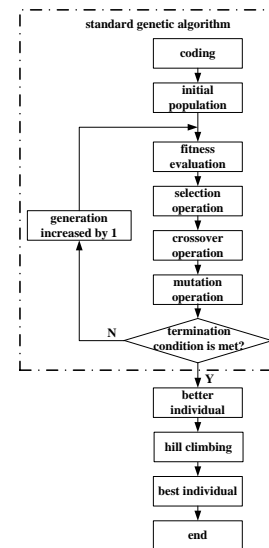

FIG. 1 FIREPOWER ASSIGNMENT METHOD FOR SHIP-TO-AIR MISSILE SYSTEM BASED ON HYBRID GENETIC ALGORITHM

1) Standard Genetic Algorithm Design

Binary encoding. Binary coding handles each chromosome as cord of bits, either 1 or 0. The length of individual chromosome is $M \times N$. The chromosome can be expressed as $A_1^{k_h} A_2^{k_h} \cdots A_M^{k_h}$,

$A_i^{k_h} = (X_{i1}^{k_h} X_{i2}^{k_h} \cdots X_{iN}^{k_h})$. $X_{ij}^{k_h}$ is string k_h of the h-th generation, which represents the i-th illumination radar assigned to the j-th target.

Initial population. Standard genetic algorithm generates initial population by the random method, and each chromosome of initial population might or might not satisfy the constraints defined in Eq. (2) and Eq. (3). The following method can automatically generate initial population which satisfies the constraints. The steps are as follows: first of all, set '0' to all genes in all chromosomes in the population, then each chromosome is transferred to a matrix ranked $M \times N$. In each column, one or two genes are randomly set '1' so that the matrix satisfies the constraint defined in Eq. (3). Next, in each row, several genes are randomly set '1' so that the matrix satisfies the constraint defined in Eq. (2), and the third step is stop.

Fitness evaluation. Each individual represents a firepower assignment scheme, which should meet the illumination radar coverage constraint. This paper implements fitness evaluation based on Eq. (7). The A radar coverage constraint test method is as follows. Based on the sequence of the arriving targets into the A radar coverage, the method determines the A radar illumination sequence for the arriving targets. Which target arrives first, the target is on the first illumination. When two or more targets enter the A radar coverage simultaneously, the priority is given to the target of the highest threat value for the A radar. For the primary illumination of the A radar, it can be determined whether the first target is effectively illuminated based on Eq. (4). If not so, the corresponding gene in the chromosome for the A radar illuminating the first target is set to be '0'. When the A radar still has the task of illuminating other targets, it selects another target as the first target based on Eq. (4), and sets the corresponding gene in the chromosome for the A radar illuminating the target. If so, it should determine whether the A radar still has the task of illuminating the second target. If so, it should determine whether the second target can be effectively illuminated by the A radar based on Eq. (5) and Eq. (6), If not, the corresponding gene in the chromosome for the A radar illuminating the second target is set to be '0', when the the A radar still has the task of illuminating other targets, it selects another target as the second target based on

the above procedure. If so, it should determine whether the A radar still has the task of illuminating the third target. The above process is repeated until the firepower assignment scheme satisfies the A radar coverage constraint.

Selection operation. The selection operation is roulette wheel selection in the standard genetic algorithm.This paper adopts the selection strategy of associative proportional selection and distillate model, which selects the individual of maximal fitness value in the H individuals of each generation directly to be the individual of the next generation, while the other individuals of the next generation are engendered by the remaining H-1 individuals of the former generation through the roulette wheel selection method. This selection strategy, on the one hand, can guarantee that the most excellent individual survives to the next generation, on the other hand, it can have the avoidance that the chance to select the next generation is in great disparity because of each individual's different fitness value, which guarantees the diversity of population to a certain extent.

Crossover operation. Selecting two individuals randomly in the population as parent individuals. Suppose that they are chromosome k_{h_1} (which is defined by $A_1^{k_{h_1}} A_2^{k_{h_1}} \cdots A_M^{k_{h_1}}$) and chromosome k_{h_2} (which is defined by $A_1^{k_{h_2}} A_2^{k_{h_2}} \cdots A_M^{k_{h_2}}$) of the h-th generation. Interchanging $A_1^{k_{h_1}}$ and $A_M^{k_{h_2}}$, interchanging $A_2^{k_{h_1}}$ and $A_{M-1}^{k_{h_2}}$, interchanging $A_3^{k_{h_1}}$ and $A_{M-2}^{k_{h_2}}$, and so on, eventually two new individuals are generated.

Mutation operation. For the chromosome k_h of the h-th generation, $A_{d1}^{k_h}$ and $A_{d2}^{k_h}$ are randomly selected, then interchanged them to form two new chromosomes, $A_{e1}^{k_h}$ and $A_{e2}^{k_h}$. d1, d2, e1, e2=1,2,…,M, the value of d1, d2, e1, and e2 are different. Then the fitness values of the two newly created individuals are compared with the two parent individuals, if the same values, invalid mutation operation is considered to remove the two newly created individuals, and mutation operation is performed according to the above procedure. Otherwise, valid mutation operation is considered, and it substitutes the two newly

created individuals for the two parent individuals.

Adaptive probability of crossover and mutation. In order to improve the calculation speed and efficiency of genetic algorithms, the probability of crossover (defined by P_c) and probability of mutation (defined by P_m) can be dynamically adjusted. This paper adjusts the two values by the following equations (M. Srinivas etc, 1994).

$$P_c = \begin{cases} k_1(eval_{max} - eval_{big})/(eval_{max} - eval_{avg}) & eval_{big} \geq eval_{avg} \\ k_3 & eval_{big} < eval_{avg} \end{cases} \quad (11)$$

$$P_m = \begin{cases} k_2(eval_{max} - eval_g)/(eval_{max} - eval_{avg}) & eval_g \geq eval_{avg} \\ k_4 & eval_g < eval_{avg} \end{cases} \quad (12)$$

where $eval_g$ is the fitness value of the g-th individual, $eval_{big}$ is the bigger one of two crossover individuals, $eval_{max}$ and $eval_{avg}$ are the maximum and average fitness value of the current population respectively. g=1,2,…,H. Usually $k_1 = k_3 = 1.0$, $k_2 = k_4 = 0.5$ are taken.

Algorithm termination condition. Let ε be a small positive number, when the absolute difference between the fitness function values of two neighboring generations is smaller than ε, the standard genetic algorithm terminates.

2) Hill Climbing Design

Bit hill climbing is adopted because binary coding handles each chromosome, whose bit variation style is a variation of the order from left to right (Xiaozhen Yan etc, 2007). Let the single best individual from the above standard genetic algorithm be k_y with the length $M \times N$. The search process of the bit hill climbing is as follows.

Step one. Suppose the mutation of the d-th bit. The initial value of d is 1.

Step two. The mutation of the d-th bit of k_y inverts the bit value from '0' to '1', or '1' to '0', to obtain new individual (defined by k_x).

Step three. Based on the above fitness evaluation method, the fitness values of k_x (defined by $eval_x$), and k_y (defined by $eval_y$) can be calculated.

Step four. If $eval_x > eval_y$, k_y is replaced by k_x. Otherwise, it is back to step two, and d=d+1. So it is constantly repeated until d=$M \times N$, and the best individual can be obtained.

Example

Let ship-to-air missile system be equipped with four illumination radars, and the number of the arriving targets is 7. So M=4, N=7. R_j values are assumed shown in Table 1, and the assumed positions of the targets in the illumination radar coverage are shown in Table 2. Let T_z=10s, b_j=2, ε =10^{-4}, P_{dj}=0.8, H=50. Based on the proposed method, the firepower assignment scheme for the system is shown in Table 3.

TABLE 1 TARGET THREAT VALUE

target	1	2	3	4	5	6	7
R_j	0.47	0.43	0.85	0.85	0.89	0.68	0.65

TABLE 2 ASSUMED POSITIONS OF THE TARGETS IN THE ILLUMINATION RADAR COVERAGE

radar \ target		1	2	3	4	5	6	7
A	T_Y/s	203		208	192			227
	T_D/s	18		11	14			11
B	T_Y/s		241	222		252	185	
	T_D/s		14	9		14	14	
C	T_Y/s	184	193		224			212
	T_D/s	7	12		13			11
D	T_Y/s	199			176	203	217	
	T_D/s	12			15	7	15	

TABLE 3 FIREPOWER ASSIGNMENT SCHEME FOR SHIP-TO-AIR MISSILE SYSTEM

target	1	2	3	4	5	6	7
radar	C	B	A	D	D	B	C

Conclusions

The proposed hybrid genetic algorithm based on adaptive penalty function can solve constrained optimization problem of firepower assignment for ship-to-air missile system. The constrained optimization problem is converted into the unconstrained one through the introduction of adaptive penalty function. Afterwords, hybrid genetic algorithm is proposed by combining standard genetic algorithm with hill climbing to solve the unconstrained optimization problem, and to get global optimization results of the firepower assignment, which helps the firepower assignment decision for the system.

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